1.2 Finding Limits Graphically and Numerically

Definition of a limit:

Ex. Use your calculator to complete the table, and then use the results to estimate the limit.
Then graph the function to confirm your results.

$$\lim_{x\to -3}\frac{\sqrt{x+4}-1}{x+3}=$$

x	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9
$f(x)$						

Ex. Use your calculator to create a table, and then use the results to estimate the limit.
Then graph the function to confirm your results.

$$\lim_{x\to 0}\frac{\sin(4x)}{x}=$$

x						
$f(x)$						

Ex. Sketch the graph, and then use it to find the limit.

(a) $f(x)=\begin{cases} x^2+1, & x\neq 2 \\ 3, & x=2 \end{cases}$

$$\lim_{x\to 2} f(x)=$$

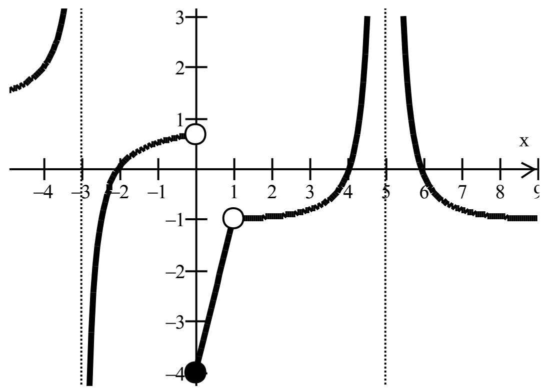
(b) $\lim_{x\to -3}\frac{|x+3|}{x+3}=$

Nonexistence of a limit:
1)

2)

3)

Ex. Use the graph of f shown on the right to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.



(a) $f(0) =$

(b) $f(1) =$

(c) $\lim_{x \to 0} f(x) =$

(d) $\lim_{x \to 1} f(x) =$

(e) $\lim_{x \to -3} f(x) =$

(f) $\lim_{x \to 5} f(x) =$

Homework: Pg. 72-74 problems 1,3,7,11,13,17,19,21,23,24,25,26,27,64

Remember that the odd-numbered answers are in the back of your textbook

1.3 Evaluating Limits Analytically

Properties of Limits:

If L, M, c, and k are real numbers and $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$, then

1. Sum Rule: $\lim_{x\to c}\left(f(x)+g(x)\right) = L+M$
2. Difference Rule: $\lim_{x\to c}\left(f(x)-g(x)\right) = L-M$
3. Product Rule: $\lim_{x\to c}\left(f(x)\cdot g(x)\right) = L\cdot M$
4. Quotient Rule: $\lim_{x\to c}\left(\frac{f(x)}{g(x)}\right) = \frac{L}{M}$ where $M \neq 0$
5. Constant Multiple Rule: $\lim_{x\to c}\left(k\cdot f(x)\right) = k\cdot L$
6. Power Rule: If r and s are integers, $s\neq 0$, then $\lim_{x\to c}\left(f(x)\right)^{r/s} = L^{r/s}$ provided that $L^{r/s}$ is a real number.
7. Limit of a Composite Function Rule: If f and g are functions such that $\lim_{x\to c} g(x) = L$ and $\lim_{x\to c} f(x) = f(L)$, then $\lim_{x\to c} f\left(g(x)\right) = f\left(\lim_{x\to c} g(x)\right) = f(L)$.

Ex. Given $f(x) = x-5$, $g(x) = x^2$. Evaluate the following.

(a) $\lim_{x\to 1} f(x) =$

(b) $\lim_{x\to 5} g(x) =$

(c) $\lim_{x\to 1} g\left(f(x)\right) =$

Evaluate.

Ex. $\lim_{x\to 2}\left(3x^2-5x+1\right) =$

Ex. $\lim_{x\to 5} \cos\left(\frac{\pi x}{6}\right) =$

Ex. $\lim_{x\to \frac{\pi}{4}} \tan(7x) =$

If we cannot evaluate a limit by direct substitution, sometimes we can use algebraic techniques to rewrite the function so that we can evaluate the limit.

Ex. $\lim_{x\to -3} \frac{x^2+x-6}{x+3} =$

Ex. $\lim_{x\to 4} \frac{\sqrt{x-3}-1}{x-4} =$

Ex. $\lim_{x\to 0} \frac{\frac{1}{x+2}-\frac{1}{2}}{x} =$

Ex. $\lim_{x\to 3} \frac{x^3-27}{x-3} =$

Hint: $a^3+b^3 =$

$a^3-b^3 =$

Ex. $\lim_{\Delta x\to 0} \frac{3(x+\Delta x)-3x}{\Delta x} =$

Ex. Find $\lim_{\Delta x\to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ given $f(x)=2x-5.$

Homework: P. 67: 12, 22, 24, 26, 32, 33, 36, 37, 40, 43, 46, 47, 50, 53, 56, 57, 59, 61, 85

1.3 Evaluating Limits Analytically (Day 2)

Use your calculator to graph $y = \frac{\sin x}{x}$.

What do you think $\lim_{x \to 0} \frac{\sin x}{x}$ equals?

Now graph $y = \frac{1 - \cos x}{x}$.

What do you think $\lim_{x \to 0} \frac{1 - \cos x}{x}$ equals?

Two special trig limits you must know:

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta} =$$

$$\lim_{\theta \to 0} \frac{1 - \cos \theta}{\theta} =$$

Ex. $\lim_{x \to 0} \frac{\sin(5x)}{5x} =$

Ex. $\lim_{x \to 0} \frac{\sin(5x)}{4x} =$

Ex. $\lim_{x \to 0} \frac{5\sin(2x)}{\sin(3x)} =$

Ex. $\lim_{x \to 0} \frac{2x + \sin x}{x} =$

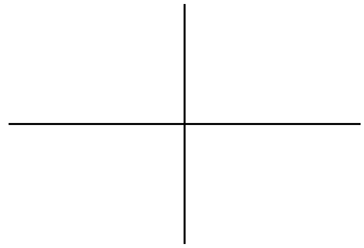
Homework: P. 67: 25, 29, 35, 38, 39, 48, 54, 55, 58, 60, 62, 65 – 75 odd, 88

<u>1.3</u> <u>More on Limits (Day 3)</u>

<u>Squeeze Theorem or Sandwich Theorem</u>:
If $h(x) \le f(x) \le g(x)$ for all x in an open interval containing c, except possibly at c itself, and if $\lim_{x\to c} h(x) = L$ and $\lim_{x\to c} g(x) = L$, then $\lim_{x\to c} f(x) = L$.

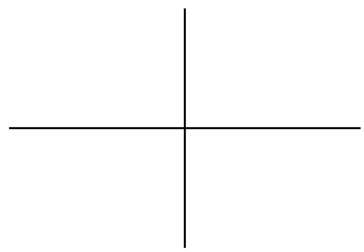
<u>Ex</u>. If $2 \le f(x) \le x^2 + 2$ for all x, find $\lim_{x\to 0} f(x)$.

Sketch a graph to illustrate.



<u>Ex</u>. Graph the given function and the equations $y = |x|$ and $y = -|x|$ in the same viewing window on your calculator. Then use the Squeeze Theorem to find $\lim_{x\to 0} f(x)$.

$$f(x) = x\sin\left(\frac{1}{x}\right)$$



<u>Homework</u>: Worksheet 1 on Limits

<u>1.3</u> <u>More on Limits (Day 4)</u>

<u>Ex.</u> Find the limit. Draw a sketch for each problem.

(a) $\lim\limits_{x\to 3^-} \dfrac{1}{x-3} =$

(b) $\lim\limits_{x\to 3^+} \dfrac{1}{x-3} =$

(c) $\lim\limits_{x\to 3} \dfrac{1}{x-3} =$

(d) $\lim\limits_{x\to 3} \dfrac{1}{(x-3)^2} =$

(e) $\lim\limits_{x\to 2^-} \dfrac{|x-2|}{x-2} =$

(f) $\lim\limits_{x\to 2^+} \dfrac{|x-2|}{x-2} =$

(g) $\lim\limits_{x\to 2} \dfrac{|x-2|}{x-2} =$

In Precalculus you learned that $y = \quad x \quad$ is called a step function or the greatest integer function. It means that y is the greatest integer that is less than or equal to x.

<u>Ex.</u> Find the limit. Draw a sketch for each problem.

(a) $\lim\limits_{x\to 1^-} \quad x-2 \quad =$

(b) $\lim\limits_{x\to 1+} \quad x-2 \quad =$

(c) $\lim\limits_{x\to 1} \quad x-2 \quad =$

Sometimes you need to use the properties of limits to make the problem easier.

<u>Ex.</u> Find the limit. Draw a sketch for each problem.

(a) $\lim\limits_{x\to 2^-} \dfrac{x^3|x-2|}{x-2} =$

(b) $\lim\limits_{x\to 2^-} \dfrac{x^3 \quad x-2}{x-2} =$

Note: Be careful when applying the properties of limits. If this approach leads to a result of 0 times ∞**,** the limit properties do not help you determine the answer. 0 times ∞ is an indeterminate form. We will study limits of indeterminate forms later in the year. On the next example, if you try to use the limit properties, you will get the wrong answer.

<u>Ex</u>. $\lim\limits_{x\to 3^+} \dfrac{x^2 - x - 6}{x-3} =$

<u>Ex</u>. $f(x) = \dfrac{2x-1}{|x|-3}$

You learned in Algebra that $|x| = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$. Use this to rewrite $f(x)$ as a piecewise function.

$f(x) = \begin{cases} \\ \\ \end{cases}$

Sketch the graph of $f(x)$ and find:

(a) $\lim\limits_{x\to\infty} f(x) =$

(b) $\lim\limits_{x\to -\infty} f(x) =$

(c) the horizontal asymptotes of $f(x)$

<u>Homework</u>: Worksheet 2 on Limits

More Practice on Limits

Ex. $\lim_{x\to 0} \dfrac{(x-5)^2-25}{x} =$

Ex. $\lim_{x\to 0} \dfrac{\frac{1}{4+x}-\frac{1}{4}}{x} =$

Ex. $\lim_{x\to 0} \dfrac{\sqrt{25+x}-5}{x} =$

Ex. $\lim_{x\to -3} \dfrac{x^2-2x-15}{x^3+27} =$

Ex. $\lim_{x\to 2^-} \dfrac{3x|x-2|}{x-2} =$

Ex. $\lim_{x\to 2^-} \dfrac{3x\ \ x-2}{x-2} =$

Ex. $\lim_{x \to 0} \dfrac{\sin x}{5x^2 - 8x} =$

Ex. $\lim_{x \to 0} \dfrac{5\sin(2x)}{\sin(3x)} =$

Homework: Worksheet 3 on Limits