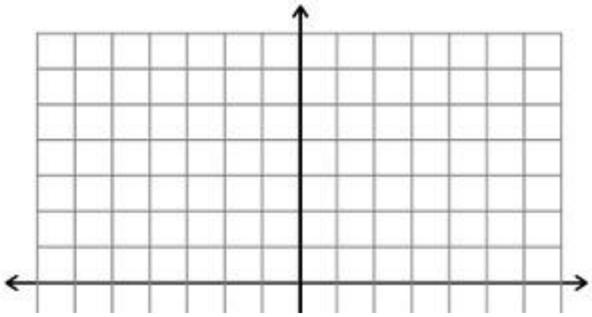
<u>Pre-AP Precalculus</u>

Module 1

Graphing and Solving Equations

1.1 The Distance & Midpoint Formulas

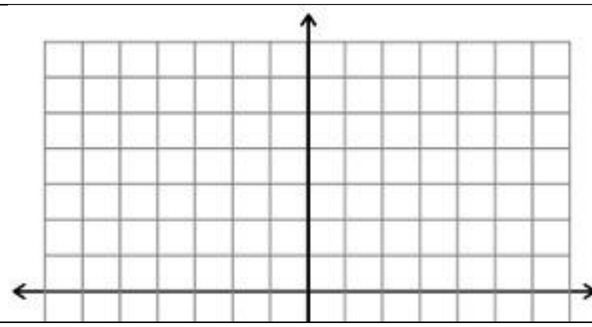
Warm Up: Find the distance d between the points $(-5, 8)$ and $(4,6)$.



The Distance Formula	This is the distance, d, between two points (x_1, y_1) and (x_2, y_2). Also denoted as $d(P_1, P_2)$ for points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$.

Example 1: Find the length of the line segment connecting $(-4,5)$ and $(3,2)$.

Example 2: Consider the three points $A = (-2,1)$, $B = (2,3)$, and $C = (3,1)$.

a) Plot each point and form the triangle ABC.	
b) Find the length of each side of the triangle.	
c) Verify that the triangle is a right triangle.	
d) Find the area of the triangle.	

1.1 The Distance & Midpoint Formulas

The Midpoint Formula	Think of the midpoint as the "____________________" of two points, P_1 and P_2, where $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$

Example 3: Find the midpoint of a line segment from $P_1 = (-5,5)$ to $P_2 = (3,1)$.

PERSONAL PRACTICE: Complete the following exercises on a separate sheet of paper. Work must be shown to receive credit for your homework.

1. Determine in which quadrant or on which axis each point lies.
   a. $(-3,2)$
   b. $(6,0)$
   c. $(-2,-2)$
   d. $(6,5)$
   e. $(0,-3)$
   f. $(6,-3)$
2. Determine in which quadrant or on which axis each point lies.
   a. $(1,4)$
   b. $(-3,-4)$
   c. $(-3,4)$
   d. $(4,1)$
   e. $(0,1)$
   f. $(-3,0)$

For exercises 3-6, find the distance $d(P_1, P_2)$ between the points P_1 and P_2.

3. $P_1 = (3,-4); P_2 = (5,4)$
4. $P_1 = (-5,-3); P_2 = (11,9)$
5. $P_1 = (4,-3);\ P_2 = (6,4)$
6. $P_1 = (a,b); P_2 = (0,0)$

1.1 The Distance & Midpoint Formulas

For exercises 7-8, plot each point and form the triangle ABC. Verify that the triangle is a right triangle. Find its area.

7. $A = (-2,5); B = (1,3); C = (-1,0)$
8. $A = (-5,3); B = (6,0); C = (5,5)$

For exercises 9-12, find the midpoint of the line segment joining the points P_1 and P_2.

9. $P_1 = (3,-4);\ P_2 = (5,4)$
10. $P_1 = (-5,-3); P_2 = (11,9)$
11. $P_1 = (4,-3); P_2 = (6,1)$
12. $P_1 = (a,b);\ P_2 = (0,0)$

For exercises 13-14, determine whether the given points are on the graph of the equation.

13. Equation: $y = x^4 - \sqrt{x}$; Points: $(0,0); (1,1); (-1,0)$
14. Equation: $y^2 = x^2 + 9$; Points: $(0,3); (3,0); (-3,0)$

Applications

15. A major league baseball "diamond" is actually a square, 90 feet on a side. What is the distance directly from home plate to second base (the diagonal of the square)?
16. Overlay a rectangular coordinate system on a major league baseball diamond so that the origin is at home plate, the positive x-axis lies in the direction from home playe to first base, and the positive y-axis lies in the direction from home plate to third base.
    a. What are the coordinates of first base, second base, and third base?
    b. If the right fielder is located at $(310, 15)$, how far is it from there to second base?
    c. If the center fielder is located at $(300,300)$, how far is it from there to third base?
17. A Ford Focus and a Mack truck leave an intersection at the same time. The Focus heads east at an average speed of 30 miles per hour, while the truck heads south at an average speed of 40 miles per hour. Find an expression for their distance apart d (in miles) at the end of t hours.
18. A hot-air balloon, headed due east at an average speed of 15 miles per hour at a constant altitude of 100 feet, passes over an intersection. Find an expression for its distance d (measured in feet) from the intersection t seconds later.
19. If $A = (-1,8)$, $M = (2,3)$, and M is the midpoint of $\overline{AB}$, find the coordinates of B.

1.2 Graphing Equations

Warm Up: Determine if the following points are on the graph of the equation $2x - y = 6$.

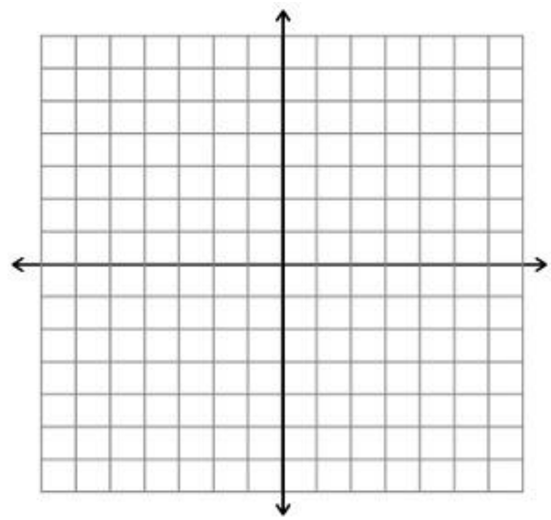
a) $(2,3)$

b) $(2,-2)$

<u>Graphing by Hand</u>

Example 1: Graph the equation: $y = -2x + 3$.

x	$y = -2x + 3$	(x, y)
-2		
-1		
0		
1		
2		



<u>Graphing by Calculator</u>

Example 2: Solve for y:

$$2y + 3x - 5 = 4.$$

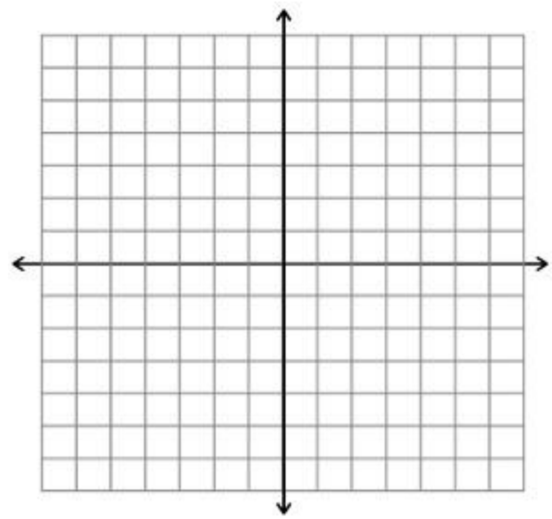
Yes…we have constant access to calculators that can graph relations – but some functions this year <u>can not</u> be graphed that way. So you may want to practice rearranging equations by hand again. ¯_(ツ)_/¯

Example 3: Use a graphing calculator to graph the equation: $6x^2 + 3y = 36$ and create a table that displays the points for $x = -3, -2, -1, 0, 1, 2,$ and 3.

1.2 Graphing Equations

Finding Intercepts		
x-intercepts		
y-intercepts		

Example 4: Find the x-intercept(s) and the y-intercept(s) of the graph of $y = x^2 - 4$. Then graph $y = x^2 - 4$ by plotting points.



Example 11: Use a graphing calculator to approximate the intercepts of the equation $y = x^3 - 16$.

PERSONAL PRACTICE: Complete the following exercises on a separate sheet of paper. Work must be shown to receive credit for your homework.

For exercises 1-2, determine whether the given points are on the graph of the equation.

1. Equation: $y = x^4 - \sqrt{x}$; Points: $(0,0)$; $(1,1)$; $(-1,0)$
2. Equation: $y^2 = x^2 + 9$; Points: $(0,3)$; $(3,0)$; $(-3,0)$

1.2 Graphing Equations

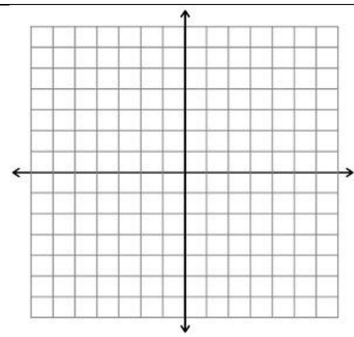
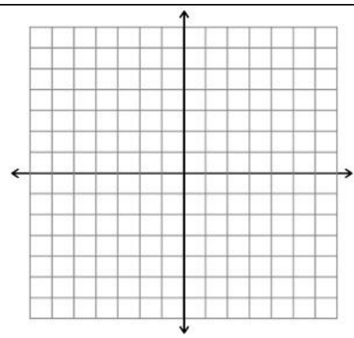
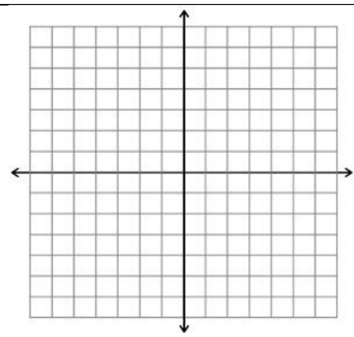
For exercises 3-6, use a graphing calculator to approximate the intercepts rounded to three decimal places. If necessary, use the table to establish an appropriate viewing window.

3. $y = 2x - 13$
4. $3x - 2y = 43$
5. $5x^2 + 3y = 37$
6. $y = 2x^2 - 1$

In exercises 7-12, find the intercepts of each equation by hand.

7. $y = x + 2$
8. $y = 2x + 8$
9. $y = x^2 - 1$
10. $y = -x^2 + 4$
11. $2x + 3y = 6$
12. $9x^2 + 4y = 36$

1.3 Graph Symmetry & Solutions

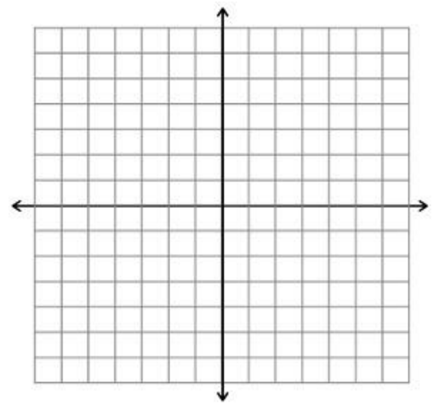
Types of Symmetry		
Vertically Reflectional		$(x, y) \rightarrow$ (_______, _______) Replace _____________ with ____________ and simplify to find an equal equation.
Horizontally Reflectional		$(x, y) \rightarrow$ (_______, _______) Replace _____________ with ____________ and simplify to find an equal equation.
180° Rotational		$(x, y) \rightarrow$ (_______, _______) Replace _____________ with ____________ and simplify to find an equal equation.

Finding Symmetry

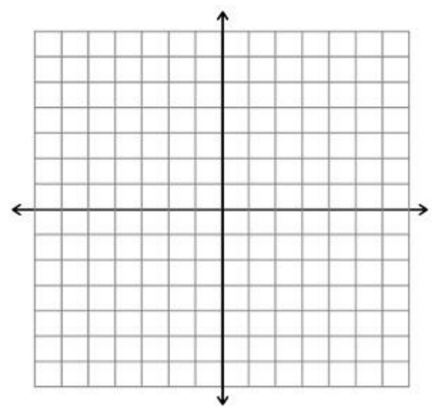
Example 1: Test the equation $y = \frac{x^2-4}{x^2+1}$ for symmetry and find the intercepts.

1.3 Graph Symmetry & Solutions

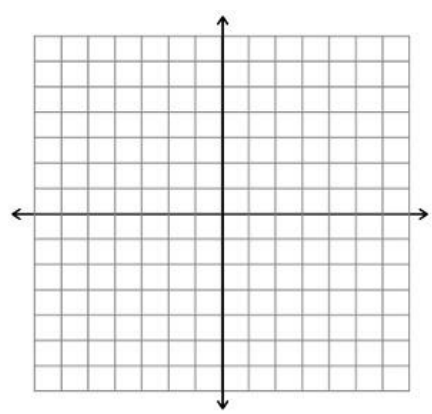
Example 2: Test the equation $y = x^3$ for symmetry, find the intercepts and graph by hand.



Example 3: Test the equation $x = y^2$ for symmetry. Find any intercepts and graph $x = y^2$, where $y \geq 0$.



Example 4: Test the equation $y = \frac{1}{x}$ for symmetry. Find any intercepts and graph.



1.3 Graph Symmetry & Solutions

Solving Equations

______________________, _____________, ____________, ___________________ - they're all the same

Using a Calculator

Example 5: Find the solution(s) of the equation $x^3 - x + 1 = 0$. (Remember: Round answers to three decimal places.)

Example 6: Find the solution(s) to the equation $4x^4 - 3 = 2x + 1$.

Solve for zero and graph	or	Graph both sides separately

By Hand

Example 7: Solve the equation $3(x - 2) = 5(x - 1)$ algebraically and numerically.

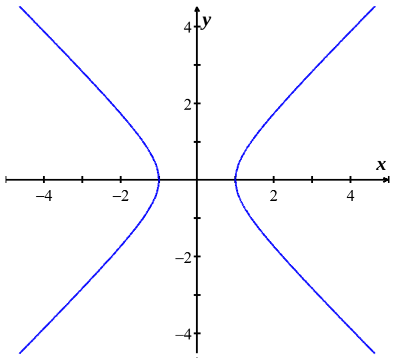
PERSONAL PRACTICE: Complete the following exercises on a separate sheet of paper. Work must be shown to receive credit for your homework.

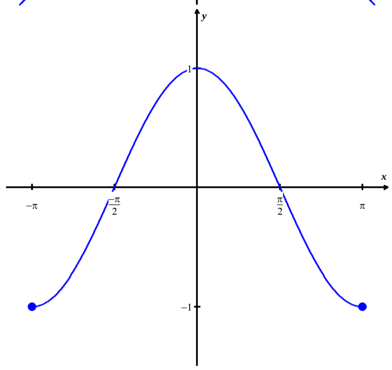
In exercises 1 – 4, determine the points that are symmetric to the given point with respect to (a) the x-axis, (b) the y-axis, and (c) the origin.

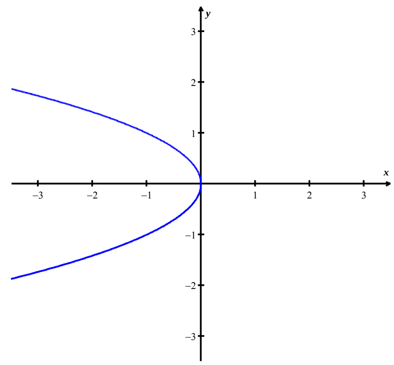
1. $(3,4)$
2. $(-2,1)$
3. $(5,-2)$
4. $(0,-3)$

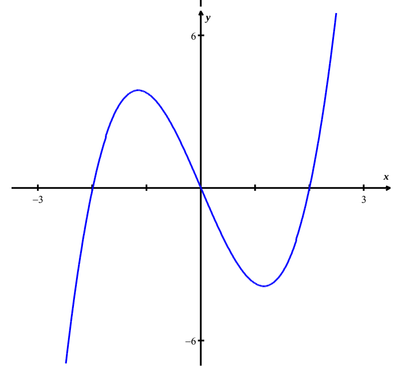
1.3 Graph Symmetry & Solutions

In exercises 5-8, the graph of an equation is given. (a) Find the intercepts. (b) Indicate whether the graph is symmetric with respect to the x-axis, the y-axis, and/or the origin.

5. 

6. 

7. 

8. 

In exercises 9-14, find the intercepts and test for symmetry.

9. $y^2 = x + 4$

10. $y = \sqrt[3]{x}$

11. $y = x^4 - 8x^2 - 9$

12. $9x^2 + 4y^2 = 36$

13. $y = x^2 - 3x - 4$

14. $y = \frac{3x}{x^2+9}$

15. If $(3, b)$ is a point on the graph of $y = 4x + 1$, what is b?

16. If $(a, 4)$ is a point on the graph of $y = x^2 + 3x$, what is a?

In exercises 17 – 20, use a graphing calculator to approximate the real solutions, if any, of each equation to three decimal places. All solutions lie between $x = -10$ and $x = 10$.

17. $x^3 - 4x + 2 = 0$

18. $-x^3 - \frac{5}{3}x^2 + \frac{7}{2}x + 2 = 0$

19. $-2x^4 + 5 = 3x - 2$

20. $-\frac{2}{3}x^4 - 3x^3 + \frac{5}{2}x = -\frac{2}{3}x^2 + \frac{1}{2}$

In exercises 21 – 40, solve each equation algebraically. Verify your solution using a graphing calculator.

21. $2(3 + 2x) = 3(x - 4)$

22. $3(2 - x) = 2x - 1$

23. $8x - (2x + 1) = 3x - 13$

24. $5 - (2x - 1) = 10 - x$

25. $\frac{x+1}{3} + \frac{x+2}{7} = 5$

1.3 Graph Symmetry & Solutions

26. $\frac{2x+1}{3} + 16 = 3x$

27. $\frac{5}{y} + \frac{4}{y} = 3$

28. $\frac{4}{y} - 5 = \frac{18}{2y}$

29. $(x+7)(x-1) = (x+1)^2$

30. $(x+2)(x-3) = (x-3)^2$

31. $x^2 - 3x - 28 = 0$

32. $x^2 - 7x - 18 = 0$

33. $3x^2 = 4x + 4$

34. $5x^2 = 13x + 6$

35. $x^3 + x^2 - 4x - 4 = 0$

36. $x^3 + 2x^2 - 9x - 18 = 0$

37. $\sqrt{x+1} = 4$

38. $\sqrt{x-2} = 3$

39. $\frac{2}{x+2} + \frac{3}{x-1} = -\frac{8}{5}$

40. $\frac{1}{x+1} - \frac{5}{x-4} = \frac{21}{4}$

1.4 Linear Functions

Slope of a Line	

SPECIAL LINES

Vertical Line:

Horizontal Line:

Example 1: What is the slope of the line containing the points $(1,2)$ and $(5,-3)$, and explain its meaning in several ways.

Types of Linear Equations		
Slope – Intercept Form		• m is the slope • $(0,b)$ is the y-intercept
Point – Slope Form		• m is the slope • (x_1,y_1) is any given point on the line
Standard Form		• $-\frac{A}{B}$ is the slope • $\left(0,\frac{C}{B}\right)$ is the y-intercept • $\left(\frac{C}{A},0\right)$ is the x-intercept
General Form		• $-\frac{A}{B}$ is the slope • $\left(0,-\frac{C}{B}\right)$ is the y-intercept • $\left(-\frac{C}{A},0\right)$ is the x-intercept
Horizontal Line		• b is a constant value • y-intercept at $(0,b)$
Vertical Line		• a is a constant value • x-intercept at $(a,0)$

1.4 Linear Functions

Example 2: **Find the Equation for**
The line with slope 4 containing the point (1,2) in
(a) point-slope form

(b) slope-intercept form.

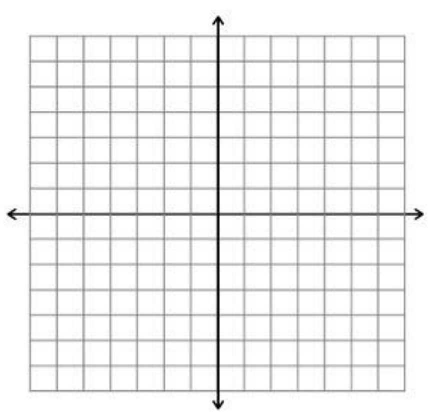
The horizontal line containing the point (3,2).

The vertical line containing the point (−5,3).

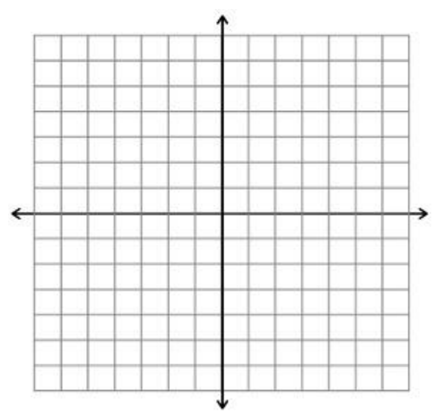
The line containing the points (2,3) and (−4,5).

Example 3: **Find the Slope and y-intercept of**
The equation $2x + 4y = 8$.

Example 4: **Graph the Equation:**
$y = -\frac{3}{5}x + 5$



$2x + 4y = 8$.



1.4 Linear Functions

Parallel v. Perpendicular	
Parallel	
Perpendicular	

Example 5: Show that the lines given by the following equations are parallel: $L_1: 2x + 3y = 6$ and $L_2: 4x + 6y = 0$.

Example 6: Find an equation for the line that contains the point $(2, -3)$ and is parallel to the line $2x + y = 6$.

Example 7: Find an equation of the line that contains the point $(1, -2)$ and is perpendicular to the line $x + 3y = 6$.

PERSONAL PRACTICE: Complete the following exercises on a separate sheet of paper. Work must be shown to receive credit for your homework.

For exercises 1 – 4, calculate the slope of the line containing each pair of points.

1. $(2,3); (4,0)$
2. $(-2,3); (2,1)$
3. $(-3,-1); (2,-1)$
4. $(-1,2); (-1,-2)$

For exercises 5 – 18, find an equation for the line with the given properties in either the standard form or slope-intercept of a linear equation.

5. Slope = 3; containing the point $(-2,3)$
6. Slope = $-\frac{2}{3}$; containing the point $(1,-1)$

1.4 Linear Functions

7. Containing the points $(1,3)$ and $(-1,2)$
8. Containing the points $(-3,4)$ and $(2,5)$
9. Slope = 3; y-intercept = 3
10. x-intercept = 2; y-intercept = -1
11. Slope undefined; containing the point $(2,4)$
12. Horizontal; containing the point $(-3,2)$
13. Parallel to the line $y = 2x$; containing the point $(-1,2)$
14. Parallel to the line $2x - y = -2$; containing the point $(0,0)$
15. Parallel to the line $x = 5$; containing the point $(4,2)$
16. Perpendicular to the line $y = \frac{1}{2}x + 4$; containing the point $(1,-2)$
17. Perpendicular to the line $2x + y = 2$; containing the point $(-3,0)$
18. Perpendicular to the line $x = 8$; containing the point $(3,4)$

For exercises 19 – 24, find the intercepts of the graph of each equation.

19. $2x + 3y = 6$
20. $-4x + 5y = 40$
21. $\frac{1}{2}x + \frac{1}{3}y = 1$
22. $x - \frac{2}{3}y = 4$
23. $0.2x - 0.5y = 1$
24. $-0.3x + 0.4y = 1.2$

For exercises 25 – 28, answer the following geometry applications.

25. Use slopes to show that the triangle whose vertices are $(-2,5), (1,3)$, and $(-1,0)$ is a right triangle.
26. Use slopes to show that the quadrilateral whose vertices are $(1,-1), (4,1), (2,2)$, and $(5,4)$ is a parallelogram.
27. Use slopes to show that the quadrilateral whose vertices are $(-1,0), (2,3), (1,-2)$, and $(4,1)$ is a rectangle.
28. Use slopes and the distance formula to show that the quadrilateral whose vertices are $(0,0)$, $(1,3)$, $(4,2)$, and $(3,-1)$ is a square.

1.5 Circular Equations

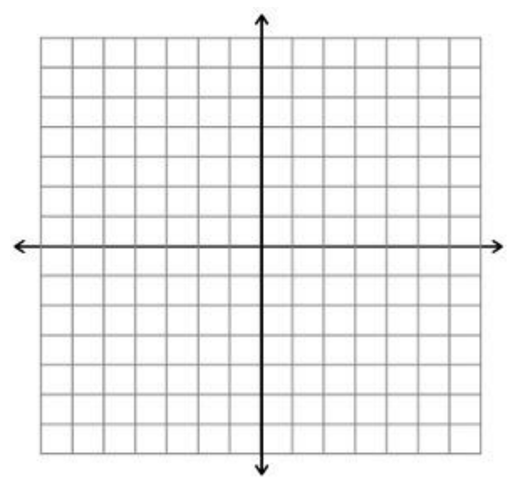
<table>
<tr><td>The Standard Form of a Circle</td><td rowspan="4">• r = radius
• (h, k) = center of the circle

Center at the Origin?
$x^2 + y^2 = r^2$
Radius of 1?
$x^2 + y^2 = 1$ *

*This is called a “unit circle” with a radius of 1 unit</td></tr>
<tr><td></td></tr>
<tr><td>The General Form of a Circle</td></tr>
<tr><td></td></tr>
</table>

Example 1: Write the standard form of the equation of the circle with radius 5 and center $(-3,6)$.

Example 2: Identify the center and radius of the circular equation $(x-3)^2 + (y+2)^2 = 16$, and graph the circle.

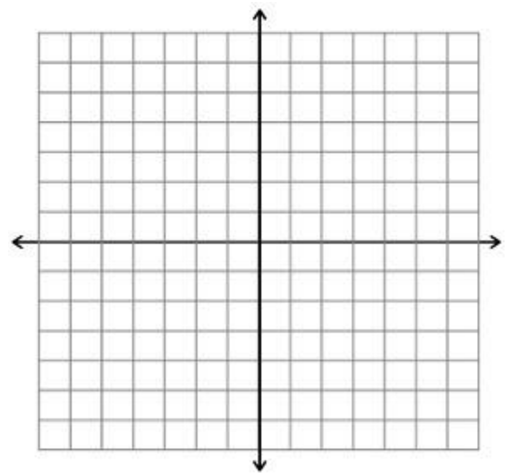


Example 3: For the circle $(x-3)^2 + (y+2)^2 = 16$, find the intercepts, if any, algebraically.

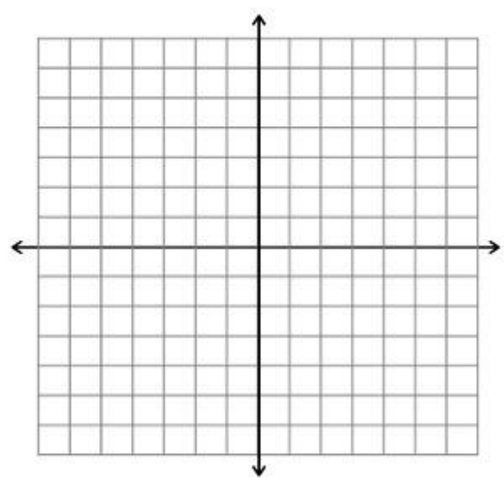
1.5 Circular Equations

Complete the Square	
	• Divide b by 2 and square it. • Add the results to both sides.

Example 15: Graph the equation $x^2 + y^2 + 4x - 6y + 12 = 0$.



Example 16: Find the general equation of the circle whose center is $(1, -2)$ and whose graph contains the point $(4, -2)$.



1.5 Circular Equations

PERSONAL PRACTICE: Complete the following exercises on a separate sheet of paper. Work must be shown to receive credit for your homework.

For exercises 1-4, write the standard form of the equation of each circle of radius r and center (h,k).

1. $r = 2; (h,k) = (0,0)$
2. $r = 5;\ (h,k) = (4,-3)$
3. $r = 4;\ (h,k) = (-2,1)$
4. $r = \frac{1}{2};\ (h,k) = \left(\frac{1}{2},0\right)$

For exercises 5-10, find the (a) center, (b) radius, and (c) intercepts of each circle.

5. $3(x+1)^2 + 3(y-1)^2 = 6$
6. $x^2 + y^2 - 2x - 4y - 4 = 0$
7. $x^2 + y^2 + 4x - 4y - 1 = 0$
8. $x^2 + y^2 - x + 2y + 1 = 0$
9. $2x^2 + 2y^2 - 12x + 8y - 24 = 0$
10. $2x^2 + 8x + 2y^2 = 0$

1.6 Solving Quadratics

For exercises 11-18, find the standard form equation of each circle.

11. Center at the origin and containing the point $(-2,3)$
12. Center $(1,0)$ and containing the point $(-3,2)$
13. Center $(2,3)$ and tangent to the x-axis
14. Center $(-3,1)$ and tangent to the y-axis
15. With endpoints of a diameter at $(1,4)$ and $(-3,2)$
16. With endpoints of a diameter at $(4,3)$ and $(0,1)$
17. Center $(-1,3)$ and tangent to the line $y = 2$
18. Center $(4,-2)$ and tangent to the line $x = 1$

1.6 Solving Quadratics

Quadratic Equation	
	• Where $a \neq 0$

Zero Product Property

Example 1 – Solve $3x^2 - x = 10$ by factoring.

$x^2 = k$ **, for any (real) value** k	
	• $k < 0 \rightarrow$ • $k = 0 \rightarrow$ • $k > 0 \rightarrow$

Example 2 – Solve $3x^2 = 9$.

Example 3 – Solve $2(x + 4)^2 = 6$.

PERSONAL PRACTICE: Complete the following exercises on a separate sheet of paper. Work must be shown to receive credit for your homework.

In exercises 1 – 6, solve each equation by factoring.

1. $x^2 - 8x + 15 = 0$
2. $x^2 - 5x = 14$
3. $2y^2 + 5y - 3 = 0$
4. $4t^2 + 9t + 2 = 0$
5. $3u^2 + u = 4$
6. $12x^2 + 13x = 4$

In exercises 7 – 10, solve the equation by taking the square root of both sides. Give exact solutions.

7. $3x^2 = 12$
8. $-5s^2 = -30$
9. $25x^2 - 4 = 0$
10. $-3w^2 + 8 = -20$

1.6 Solving Quadratics (cont.)

Complete the Square	- Divide b by 2 and square it. - Add the results to both sides to keep the equation balanced.

Example 4 – Solve $2x^2 - 6x + 1 = 0$ by completing the square.

Example 5 – Solve $ax^2 + bx + c = 0$ by completing the square.

Quadratic Formula	You really should memorize this if you haven't already. [Bonus: it's on the SAT and TSI]

Example 6 – Solve $x^2 + 3 = -8x$ by using the quadratic formula.

1.6 Solving Quadratics (cont.)

The Discriminant	Helps determine the number of <u>real</u> solutions. • $< 0 \quad \rightarrow$ • $= 0 \quad \rightarrow$ • $> 0 \quad \rightarrow$

Example 7 – Solve $2x^2 = -x - 3$.

Example 8 – Solve $4x^4 - 13x^2 + 3 = 0$.

PERSONAL PRACTICE: Complete the following exercises on a separate sheet of paper. Work must be shown to receive credit for your homework.

In exercises 1 – 4, solve the equation by completing the square.

1. $x^2 - 2x = 12$
2. $x^2 - 4x - 30 = 0$
3. $w^2 - w - 1 = 0$
4. $t^2 + 3t - 2 = 0$

In exercises 5 – 8, use the quadratic formula to solve the equation.

5. $x^2 + 6x + 7 = 0$
6. $x^2 + 6 = 2x$
7. $4x^2 - 4x = 7$
8. $4x^2 - 8x + 1 = 0$

In exercises 9 – 14, solve the equation by any method.

9. $x^2 + 9x + 18 = 0$
10. $4x(x + 1) = 1$
11. $2x^2 = 7x + 15$
12. $t^2 + 4t + 13 = 0$
13. $\frac{7x^2}{3} = \frac{2x}{3} - 1$
14. $25x + \frac{4}{x} = 20$

In exercises 15 – 18, find a number *k* such that the given equation has exactly one real solution.

15. $x^2 + kx + 25 = 0$
16. $x^2 - kx + 49 = 0$
17. $kx^2 + 8x + 1 = 0$
18. $kx^2 + 24x + 16 = 0$

Module 1 – Selected Solutions