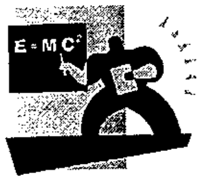


Are You Ready for Pre-Calculus and Physics? Summer Math Review Packet

This packet contains math problems, formulas, concepts, and examples that will be useful to you in Pre-Calculus and Physics. It is important that you work the problems, check your answers, and study the concepts presented. Most, if not all, of this packet should be familiar to you. Work all problems on your own paper and have this ready to turn in to your math teacher at the begnning of school. Show all work neatly and orderly.

I. Definitions- Write a brief explanation for meaning of the following:

1. the Cartesian Plane
2. A graph is in the first quadrant.
3. f(2) = 5
4. An expression is a relation.
5. An expression is a function.
6. A zero of a function is 5.
7. y is directly proportional to x (give an example)
8. The coefficient of x^2 is 6.
9. A function has only one root.
10. The x-intercept of a graph is (3,0).
11. The y-intercept of a graph is (0, -2).
12. A function is a polynomial (give an example).
13. The graph of f(x) has no x-intercepts.
14. The graph of f(x) has no y-intercept.
15. $f^{-1}(x)$
16. If (x, y) is a point on the graph of f(x), then (- x, y) is also a point on the graph.
17. If (x, y) is a point on the graph of f(x), then (x, - y) is also a point on the graph.
18. If (x, y) is a point on the graph of f(x), then (-x, -y) is also a point on the graph.

II. Formulas- State the following formulas:

1. Distance Formula (distance between two points (x_1, y_1) *and* (x_2, y_2))
2. Midpoint Formula (for the midpoint of the line segment connecting points (x_1, y_1) *and* (x_2, y_2))

*Find AB (length) and the midpoint of segment AB if A = (-1, 2) and B = (5,4)

3. Quadratic Formula for finding x in $y = ax^2 + bx + c$
4. Pythagorean Theorem
5. Relationship between the sides in any isosceles right triangle (45°-45°-90°) if a leg has length x.

* If the length of the hypotenuse = 12, find the lengths of the legs.

6. Relationship between the sides in a 30°-60°-90° triangle with the short leg of length x.

* If the length of the long leg = 9, find the lengths of the hypotenuse and the short leg.

7. Standard form for the equation of a circle with center (h,k) and radius r

*Find the standard form of the equation of a circle with center (3, -2)and solution point (-1,1).

8. Slope of a line through the points (x_1, y_1) *and* (x_2, y_2)

* Find the slope of the line through the points A and B in #2 above.

9. Vertex of the graph of $f(x) = ax^2 + bx + c$ *State the vertex of the graph of $y = 2x^2 + 4x - 3$.
10. Standard from of a quadratic function (also called (h,k) form)

    *Change the function to (h,k) form: $y = 2x^2 + 4x + 4$

III. Linear equation forms-

"point-slope form" $y - y_1 = m(x - x_1)$
"slope-intercept form" $y = mx + b$
"standard form" $Ax + By + C = 0$
vertical line $x = a$
horizontal line $y = b$

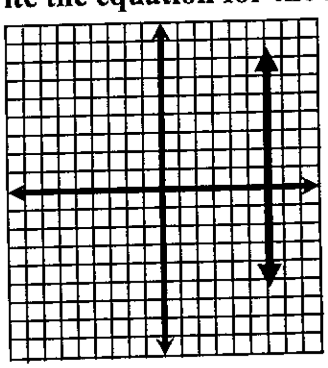
Parallel lines have the same slope. Perpendicular lines have slopes that are negative reciprocals of each other.

Write an equation of a line based on the given information. (You may leave your equation in any form unless a form is specifically asked for.)

1. Find the equation of the line that has a slope of 5 and passes through the point (3, -4).
2. Find the equation of the line that passes through the points (4,1) and (3, -2).
3. Find the equation of the line that passes through the point (-2, 1) and is parallel to the line 4x + 2y = -1.
4. Find the equation of the line that has a slope of 0 and passes through the point (-5, 1).
5. Find the equation of the line that passes through the origin and is perpendicular to the line 3x + 4y = -7.
6. Find the equation of the line that has an undefined slope and passes through the point (4, -5).
7. Find the equation of the line that has an x-intercept of 5 and a y-intercept of 3.
8. Find the equation of the vertical line that passes through the point (3,2).
9. Find the equation of the horizontal line that passes through the point (1, -5).
10. Find the x- and y-intercepts of the line that passes through the point (3, -5) and is perpendicular to the line $x - 3y = -2$.

Write the equation for the following graphs:

11.



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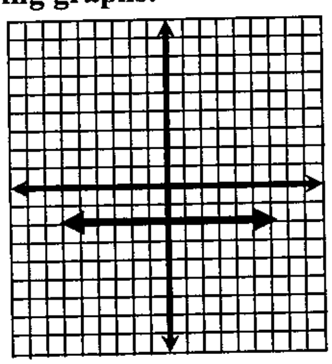

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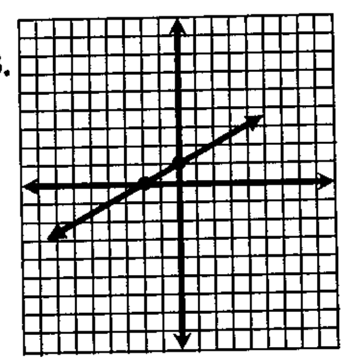


14. The table lists the number of households, h (in the millions), in the US that owned computers between 1990 and 1997. Approximate the best-fitting line for this data. Let t represent the year, with t = 0 corresponding to 1990. If this pattern were to continue, how many households would own computers in 2006?

Year	1990	1991	1992	1993	1994	1995	1996	1997
Households	26	28.4	29.1	30.2	31.4	32.3	33.8	34.1

IV. Quadratic equations forms- **general form** $f(x) = ax^2 + bx + c$

standard form $f(x) = a(x - h)^2 + k$ **where** $a \neq 0$

(also can be written as $(x - h)^2 = 4p(y - k)$ **where** $a = 1/(4p)$ **)**

Solve the following problems:

1. A baseball is hit at a point 3 feet above the ground at a velocity of 100 feet per second and an angle of 45°. The path of the ball is given by the function $f(x) = -0.0032x^2 + x + 3$, where x and y are measured in feet. Will the baseball clear a 10-foot fence located 300 feet from home plate? Justify your answer.

2. A zookeeper has 500 ft. of fencing and wants to build a rectangular pen. Find a quadratic equation that relates the area (A) of the pen to its length (l). Determine the length that will give the maximum area.

3. A company determines that it can sell 100 calculators per month if the selling price is $25. For each $2 decrease in the price, the number of calculators sold per month increases by 20. Find the revenue function and determine the price the company should sell the calculators for if they want revenue of $3000 for the month.

V. Factoring- Factor each of the following:

1. $x^2 - 16$
2. $x^2 - x - 6$
3. $6x^2 - x - 2$
4. $4x^3 - 19x^2 - 5x$
5. $x^2 + 9$
6. $x^3 + 27$
7. $x^3 - 8$
8. $8x^6 - 125$
9. $(2x-3)^3(x+1) + (x-3)(2x-3)^2$
10. $(3x-2)^{-4}(x+3) + (x+3)^2(3x-2)^{-3}$

VI. Finding the roots of a function-

Finding "zeros" or "roots" or "solutions" of a function f(x) means setting f(x) = 0 and solving. Synthetic division is a useful tool for finding zeros or roots of some polynomials. The quadratic formula will always find roots of a quadratic (2nd degree) equation.

Find solutions for the following functions:

1. By factoring: $x^2 + 5x - 24 = 0$
2. By factoring: $x^3 + 5x = 9x$
3. By factoring: $3x^2 = 5x + 2$
4. By the quadratic formula: $6x^2 - 3x - 2 = 0$
5. Using synthetic division: $x^3 - 2x^2 - 29x + 30 = 0$
6. Use the calculator: $x^3 + x^2 - x = 0$
7. $\sqrt{x} + 1 = 41$
8. $\sqrt[3]{x+1} - 4 = -1$
9. $\dfrac{2x-1}{(x+2)(x^2+3)} = 0$
10. $x^{\frac{4}{3}} = 81$

11. $x^{-1} = -3$

12. $\frac{8+x}{x} - 5 = 0$

13. $(x - 3)^2 + 9 = 25$

14. $\frac{x}{x+2} - \frac{2}{2x-1} = \frac{1}{5}$

15. $(2x-1)^2(x-5)^2 + (2x-1)^3(x-5) = 0$

Solve the systems:

16. $y = x + 1$ and $3y - x = 5$

17. $y - x = 7$ and $y = x^2 + 2x + 5$

Solve the inequalities:

18. $2x + 1 > 6x - 9$

19. $x^2 - 2x - 3 \geq 0$

VII. Scientific Notation- Write the following numbers in scientific notation.

1. 20,000
2. 543.6
3. 0.000005
4. 34.1
5. 0.00055
6. 0.00345
7. 40,230,000
8. 628,000
9. $34.5 \text{ X } 10^5$
10. $0.004 \text{ X } 10^2$
11. $180 \text{ X } 10^{-1}$
12. $0.72 \text{ X } 10^{-2}$

VIII. Multi-variable formula manipulation- Solve each of the following equations for h. Then, calculate the value of h if g = 12, k = 0.4, and m = 1.5.

1. $kh = \frac{g}{m}$

2. $gh - k = m$

3. $\frac{g - m}{h} = k$

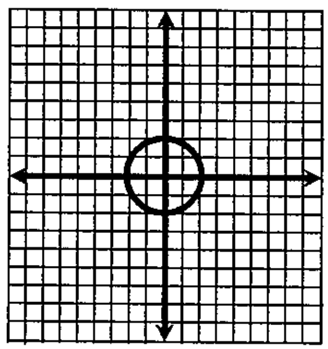
4. $\frac{mk}{g + h} = 2$

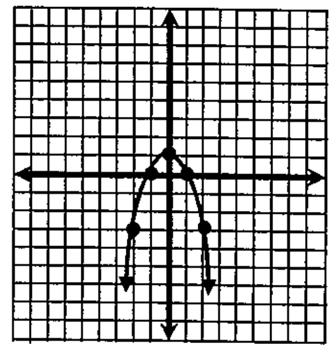
IX. **<u>Graphing-</u> Graph each of the following on graph paper. State the <u>domain</u> and <u>range</u> for each.**

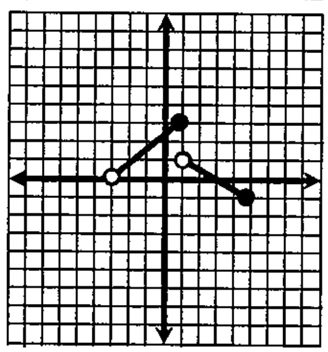
Use <u>interval notation</u> to state the domain(x values) and the range (y values).
open interval : (a, b) $a < x < b$ **"values between a and b"**
closed interval: [a, b] $a \le x \le b$ **" values between and including a and b"**
half-open (or half-closed) interval: (a, b] $a < x \le b$ **or [a, b)** $a \le x < b$
"values between a and b and including one of the endpoints"
For $x > 0$ **: (0, ∞)** **For** $x < 0$ **: (- ∞, 0)** **For** $x \ge 0$ **: [0, ∞)** **For** $x \le 0$ **: (- ∞, 0]**

1. $6x + 3y = 9$
2. $y = x^2$
3. $y = |x|$
4. $y = \sqrt{x}$
5. $y = -x^2 + 4$ For # 5-7, label the intercepts and the vertex on your graphs.
6. $y = x^2 + 2x - 3$
7. $y = 3x^2 + 5x - 2$
8. $y = \dfrac{x+1}{x-2}$
9. $y = \dfrac{-1}{x}$
10. $y = (x+1)^2$
11. $y = \begin{cases} -2, & \text{for } x < -1 \\ x, & \text{for } -1 \le x < 1 \\ 0, & \text{for } x \ge 1 \end{cases}$
12. $y = \sqrt{x-4}$

State the equation for each graph:

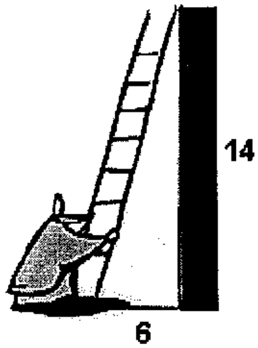
13. 

14. 

15. 

X. <u>Trigonometry-</u> Solve the following problems:

1.



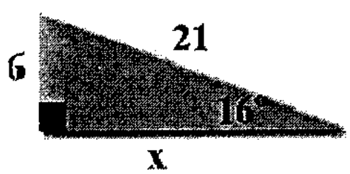
A ladder leans against a building. The foot of the ladder is 6 feet from the building. The ladder reaches height of 14 feet on the building.

a. Find the length of the ladder to the nearest foot.

____

b. Find to the nearest degree, the angle the ladder makes with the ground.

2.



Which statement can NOT be used to find the length of x?

Choose:

- $\tan 16 = \frac{6}{x}$
- $\cos 16 = \frac{x}{21}$
- $\tan 74 = \frac{x}{6}$
- $\tan 16 = \frac{x}{6}$

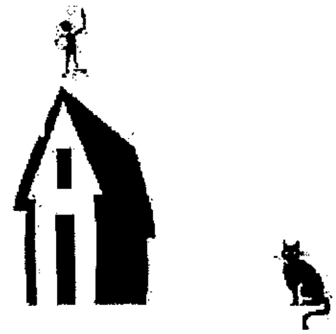
3. From a point on the ground 25 feet from the foot of a tree, the angle of elevation of the top of the tree is 32°. Find to the nearest foot, the height of the tree.

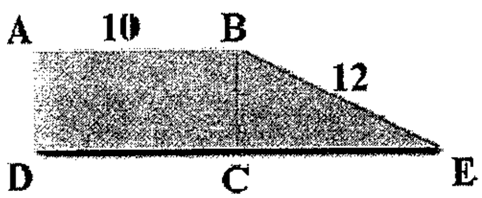
4.

The figure shown on the left is a trapezoid. Using the information given, find the area of this trapezoid to the nearest square unit.

5.

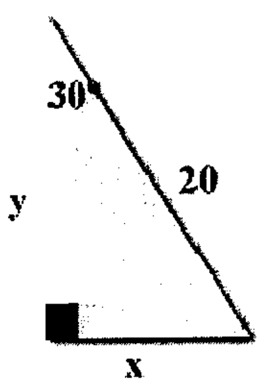
From the top of a barn 25 feet tall, you see a cat on the ground. The angle of depression of the cat is 40°. How many feet, to the nearest foot, must the cat walk to reach the barn?





In the figure on the left, ABCD is a rectangle whose perimeter is 30. The length of BE is 12. Find to the nearest degree, the measure of angle E.

7.



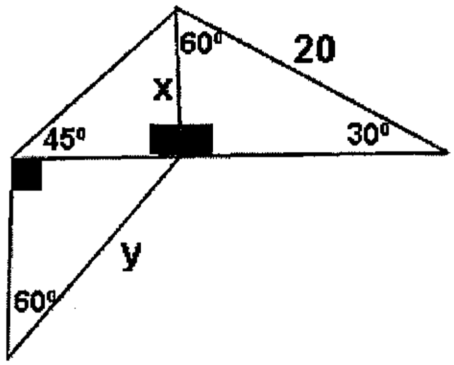
a. Find x. __________

b. Find y. __________

8.

a. Find x. __________

b. Find y.



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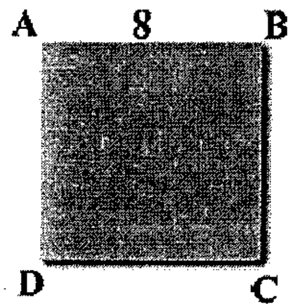
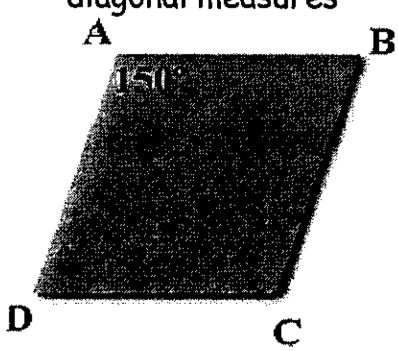


In the figure on the left, ABCD is a square whose side is 8 units. Find the length of diagonal AC to the nearest tenth.

Answer

10. In the figure below, ABCD is a rhombus. The measure of angle A is 150°. Draw the diagonals so that they intersect at E. The shorter diagonal measures

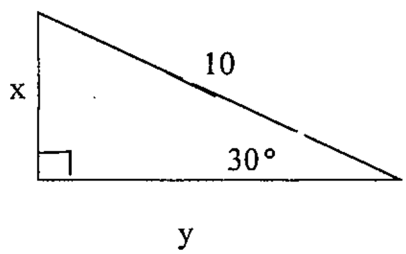


a. What is the measure of angle DEC?
b. What is the length of the sides of the rhombus to the nearest integer?
c. What is the length of the longer diagonal to the nearest integer?
d. Using your answer from part c and the given diagonal length of 10, find the area of the rhombus.

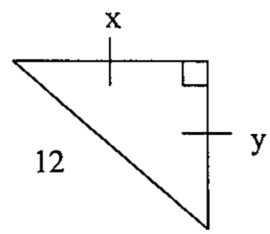
Answer

XI. <u>Special Triangles-</u> Use your knowledge of special triangles to find x and y for each of the following:

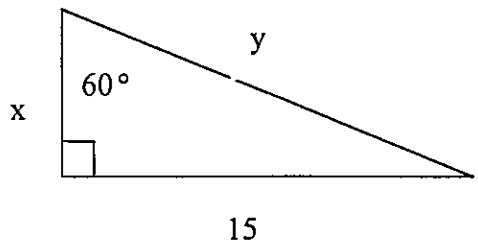
1.



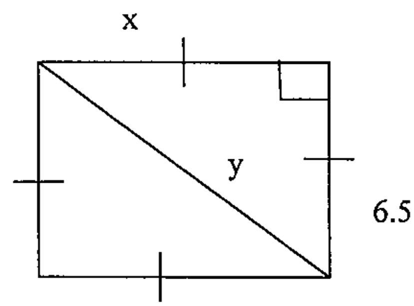
2.



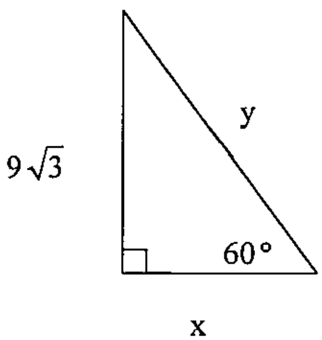
3.



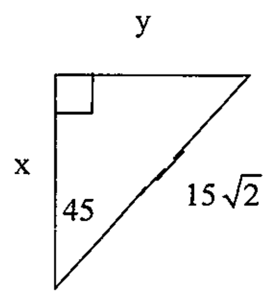
4.



5.



6.



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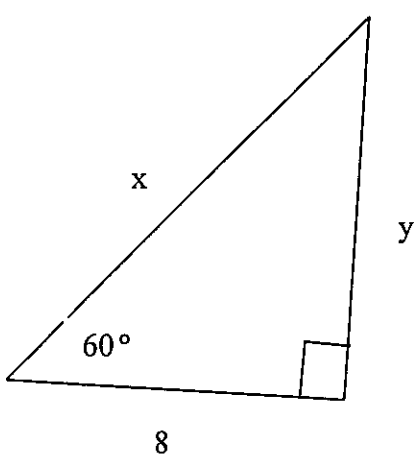

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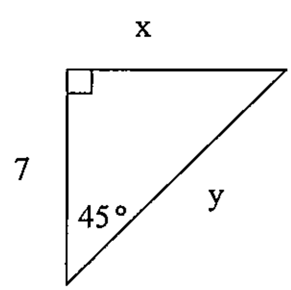

Solutions:

Part I.

1. The rectangular coordinate system used to represent ordered pairs of real numbers by points in a plane.
2. all points (x,y) such that $x > 0$ and $y > 0$
3. point (2,5)
4. A correspondence between two sets, represented as ordered pairs.
5. A relation where for each x there is one and only one y.
6. When $y = 0$, $x = 5$; intercept is (5,0)
7. $y = kx$ for some constant k
8. $6x^2$ is the term
9. one solution for $y = 0$
10. graph crosses the x-axis at 3
11. graph crosses the y-axis at -2
12. such as $3x^4 + 2x^3 + x - 2 = y$
13. the graph never crosses the x-axis; no solution to $y = 0$
14. No solution for y when $x = 0$
15. the inverse function of f(x)
16. function is symmetric with respect to the y-axis (even function)
17. function is symmetric with respect to the x-axis (not a function)
18. function is symmetric with respect to the origin (odd function)

Part II.

1. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
2. midpoint = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ *AB = $2\sqrt{10}$, (2,3)
3. x = $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
4. In any right triangle with legs of lengths a and b, and hypotenuse of length c, then $a^2 + b^2 = c^2$
5. x, x, $x\sqrt{2}$ Hypotenuse = $\sqrt{2}$ times the leg * $6\sqrt{2}$
6. x, $x\sqrt{3}$, 2x Hypotenuse = twice the short leg , and long leg = $\sqrt{3}$ times the short leg
   * $H = 6\sqrt{3}, SL = 3\sqrt{3}$
7. $(x-h)^2 + (y-k)^2 = r^2$ * $(x-3)^2 + (y+2)^2 = 25$
8. $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ *1/3
9. $\left(\frac{-b}{2a}, f(\frac{-b}{2a})\right)$ *(-1,-5)
10. $(x - h)^2 = 4p(y-k)$ or $y = a(x - h)^2 + k$, where $a = \dfrac{1}{4p}$ and $a \neq 0$. * $y = 2(x + 1)^2 + 2$

Part III.

1. $y + 4 = 5(x-3)$ or $y = 5x - 19$
2. $y - 1 = 3(x - 4)$ or $y + 2 = 3(x - 3)$ or $y = 3x - 11$
3. $y - 1 = -2(x + 2)$
4. $y = 1$
5. $y = 4/3\ x$
6. $x = 4$
7. $y = -3/5\,(x - 5)$
8. $x = 3$
9. $y = -5$
10. (4/3, 0), (0 , 4)
11. $x = 6$
12. $y = -2$
13. $y = .5x + 1$
14. $y = 1.125x + 26.725$; ≈ 45 million

Part IV.

1. yes; when x = 300, then y = 15 ft. and 15 > 10
2. $A(l) = l(250 - l)$; l = 125 ft.
3. $R(x) = 2500 + 300x - 40x^2$; $15 or $20

Part V.

1. $(x+4)(x-4)$ 2. $(x-3)(x+2)$ 3. $(3x-2)(2x+1)$ 4. $x(4x+1)(x-5)$ 5. Prime
6. $(x+3)(x^2-3x+9)$ 7. $(x-2)(x^2+2x+4)$ 8. $(2x^2-5)((4x^4+10x^2+25)$
9. $2(2x-3)^2(x^2-3)$ 10. $(x+3)(3x^2+7x-5)(3x-2)^{-4}$

Part VI.

1. $x = -8, 3$
2. $x = 0, -2, 2$
3. $x = -1/3, 2$
4. $x = \dfrac{3 \pm \sqrt{57}}{12}$
5. $x = 1, -5, 6$
6. $x =$ 0, 0.618, -1.618
7. $x = 1600$
8. $x = 26$
9. ½
10. 27
11. -1/3
12. 2
13. 7, -1
14. 3, -.75
15. ½, 5, 2
16. (1,2)
17. (-2, 5) and (1, 8)
18. (- ∞, 5/2)
19. (- ∞, - 1] , [3, ∞)

Part VII.

1. 2×10^4
2. 5.436×10^2
3. 5×10^{-6}
4. 3.41×10^1
5. 5.5×10^{-4}
6. 3.45×10^{-3}
7. 4.023×10^7
8. 6.28×10^5
9. 3.45×10^6
10. 4×10^{-1}
11. 1.8×10^1
12. 7.2×10^{-3}

Part VIII.

1. $h = \dfrac{g}{mk} = 20$
2. $h = \dfrac{m+k}{g} = 0.158$
3. $h = \dfrac{g-m}{k} = 26.25$
4. $h = \dfrac{mk}{2} - g = -11.7$

1. domain: all real numbers, range: all real numbers
2. D: all real numbers, R: [0, ∞)
3. D: all reals , R: [0, ∞)
4. D: [0, ∞) , R: [0, ∞)
5. D: all reals, R: (- ∞, 4] , vertex: (0,4) and intercepts (-2,0), (2,0)
6. D: all reals, R: [- 4, ∞), vertex: (-1, -4) and intercepts(-3, 0), (1, 0),(0, -3)
7. D: all reals , R: [-4.08333, ∞) , V: (-.83333, -4.083333) and intercepts (1/3, 0) , (-2, 0), (0, -2)
8. D: (- ∞, 2), (2, ∞), R: (- ∞, 1), (1, ∞), Asymptotes are x = 2 and y = 1
9. D: (- ∞, 0), (0, ∞), R: (- ∞, 0), (0, ∞), Asymptotes are x = 0 and y = 0
10. D: all reals, R: [0, ∞), V: (-1, 0) and intercepts (-1,0) and (0, 1)
11. piece-wise graph D: all reals R: [-1, 1), -2
12. D: [4, ∞), R: [0, ∞)
13. $x^2 + y^2 = 4$
14. $y = -x^2 + 1$
15. $y = \begin{cases} \dfrac{3}{4}x + 2, \textit{ for } -3 < x \le 1 \\ \dfrac{-2}{3}x + \dfrac{4}{3}, \textit{ for } 1 < x \le 4 \end{cases}$

(Use calculator to check graphs, but graph without a calculator first.)